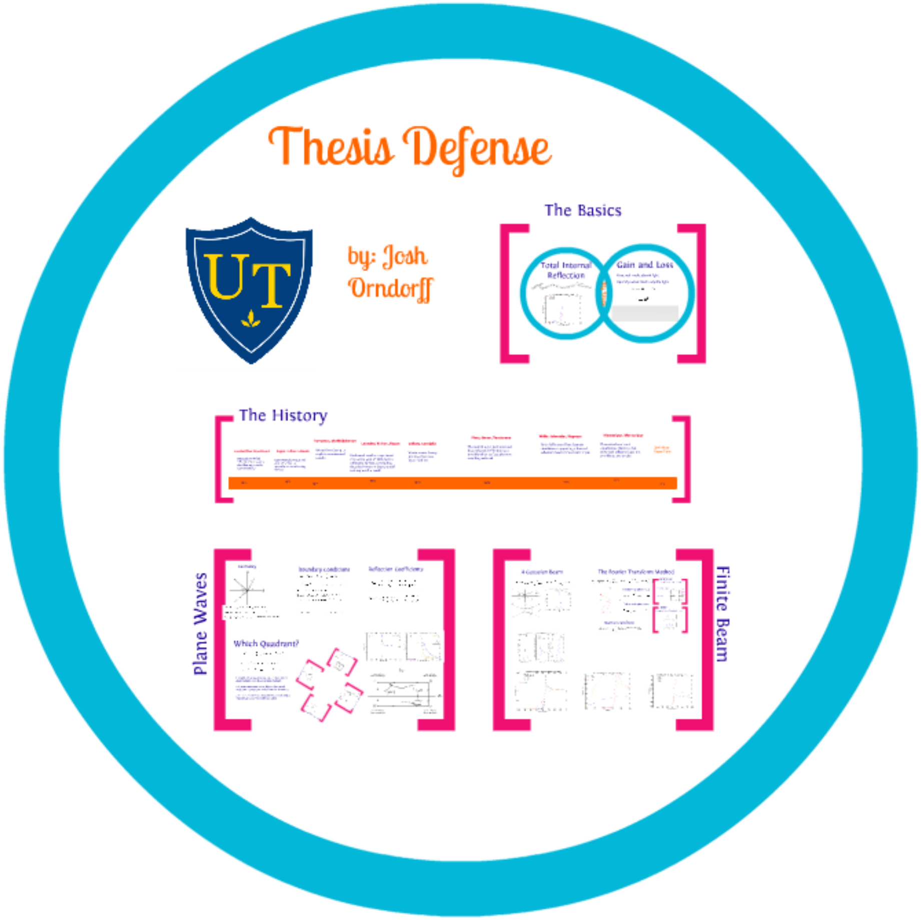
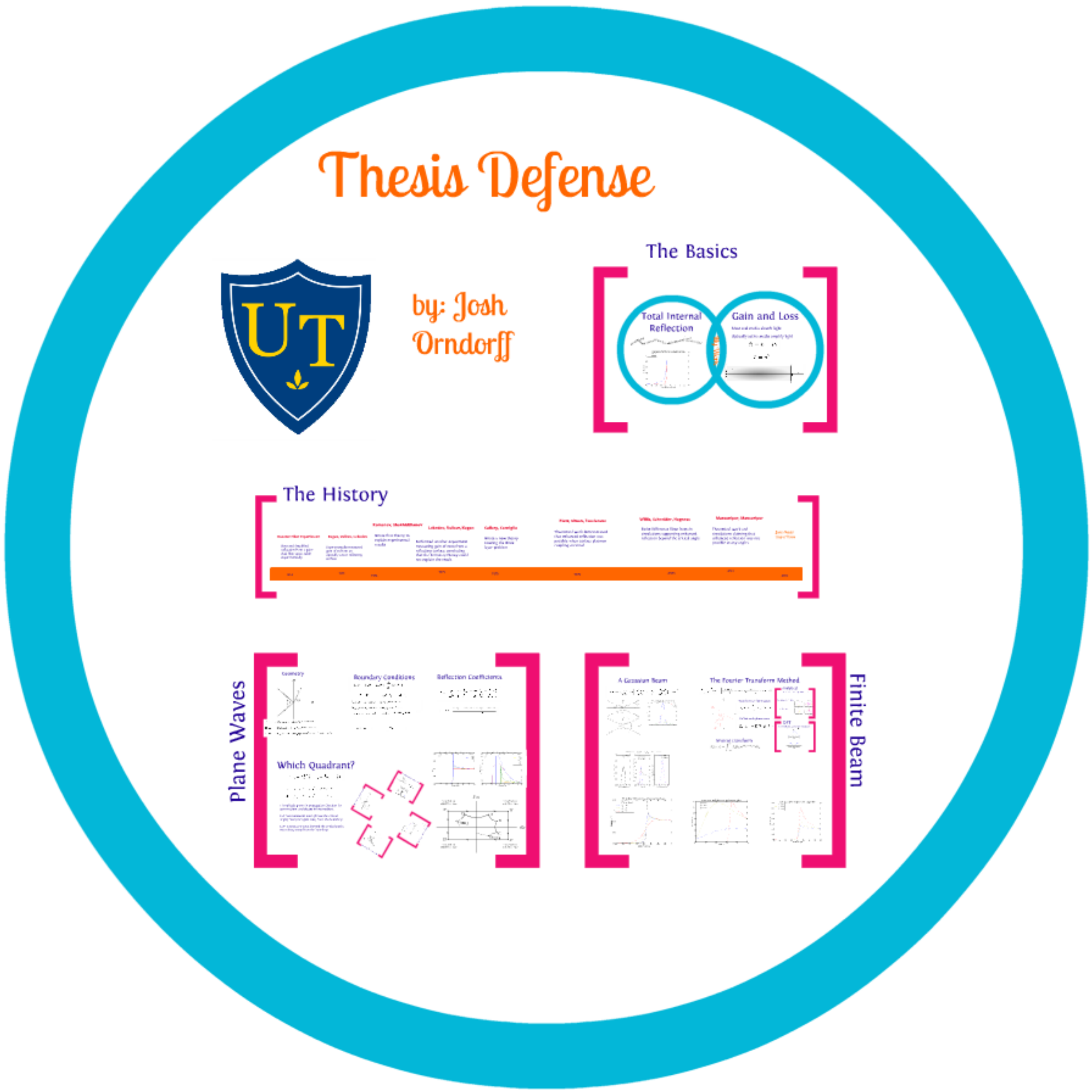
Amplified Total Internal Reflection at the Surface of a Gain Medium

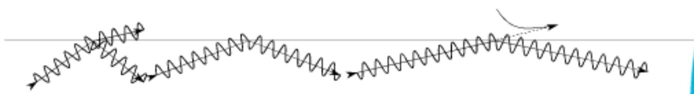


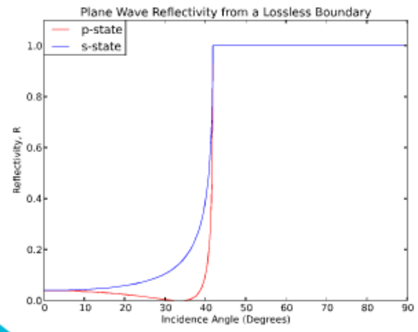
Special thanks to Dr. Deck, Dr. Karpov, and Dr. Bagley

Amplified Total Internal Reflection
at the Surface of a Gain Medium



Total Internal Reflection





My Work

Gain and Loss

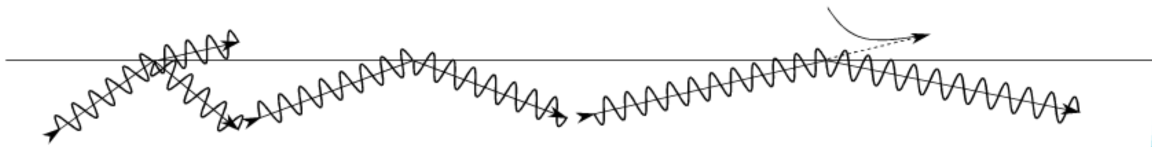
Most real media absorb light

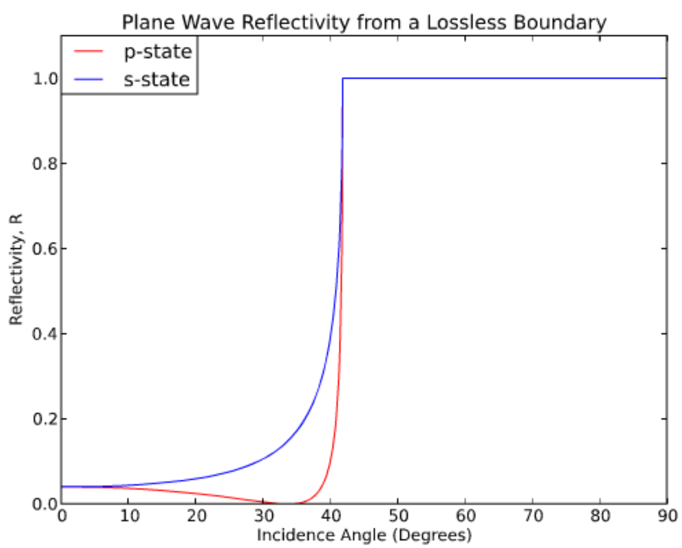
Optically active media amplify light

$$\tilde{n} = n - i\gamma$$

$$\tilde{\epsilon} = \tilde{n}^2$$

Total Internal Reflection





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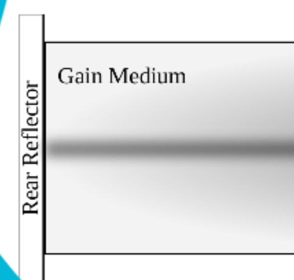

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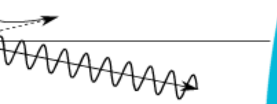

My Work

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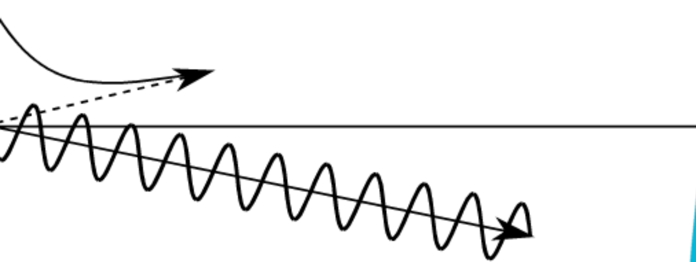

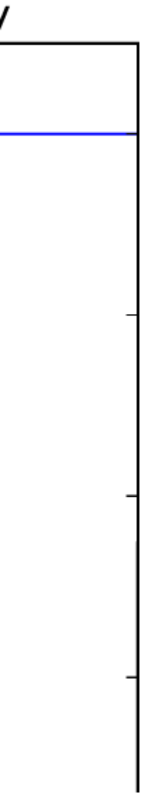

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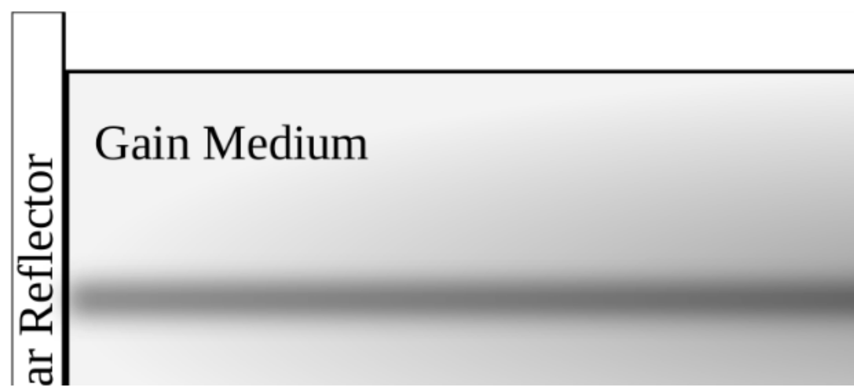


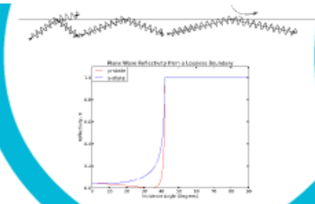
Orndorff

My Work

Optically active media amplify light

$$\tilde{n} = n - i\gamma$$

$$\tilde{\epsilon} = \tilde{n}^2$$



The History

Koester Fiber Experiment (1966)

Detected Amplified reflection from a gain-clad fiber optic cable experimentally

Kogan, Volkov, Lebedev (1972)

Experimentally measured gain of 25 from an optically active reflecting surface

Romanov, Shakhidzhanov (1972)

Wrote first theory to explain experimental results

Lebedev, Volkov, Kogan (1973)

Performed another experiment measuring gain of 1000 from a reflecting surface, concluding that the Romanov theory could not explain the result

Callary, Carniglia (1976)

Wrote a new theory treating the three-layer problem

Plotz, Simon, Tucciarone (1979)

Theoretical work demonstrated that enhanced reflection was possible when surface plasmon coupling occurred

Willis, Schneider, Hagness (2003)

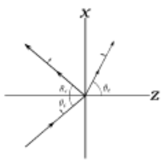
Finite Difference Time Domain simulations supporting enhanced reflection beyond the critical angle

Mansuripur, Mansuripur (2012)

Theoretical work and simulations claiming that enhanced reflection was not possible at any angles

Josh's Master Degree Thesis (2013)

Geometry

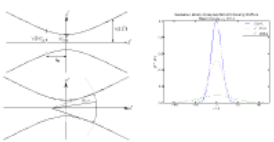


Boundary Conditions

$$k_{ix}x = k_{rx}x = k_{tx}x - \frac{\gamma\omega}{c}\sin(\theta_t)x$$

Reflection Coefficients

A Gaussian Beam



The Fourier Transform Method

Analytical

Transform to plane waves

Reflect each plane wave

$$E_r(k_x) = E_i(k_x)\tilde{r}(k_x)$$

DFT

Inverse transform

$$E_r(x,z) = \int_{-\infty}^{\infty} E_r(k_x) e^{i(k_x x + k_z z)}\, dk_x$$

Koester Fiber Experiment

Detected Amplified reflection from a gain-clad fiber optic cable experimentally

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Josh's
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Josh's Master
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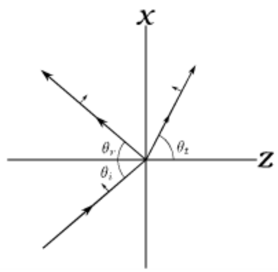
2013

Plane Waves

Geometry



$$\mathbf{E}_i(\mathbf{x},t) = E_{0i}(\cos\theta_i\hat{\mathbf{x}} - \sin\theta_i\hat{\mathbf{z}})e^{i(k_{ix}x+k_{iz}z-\omega t)}$$

$$\mathbf{E}_r(\mathbf{x},t) = E_{0r}(\cos\theta_r\hat{\mathbf{x}} + \sin\theta_r\hat{\mathbf{z}})e^{i(k_{rx}x+k_{rz}z-\omega t)}$$

$$\mathbf{E}_t(\mathbf{x},t) = E_{0t}(\cos\theta_t\hat{\mathbf{x}} - \sin\theta_t\hat{\mathbf{z}})e^{i(k_{tx}x+k_{tz}z-\omega t)}e^{\gamma\frac{\omega}{c}(\sin\theta_t x+\cos\theta_t z)}$$

Boundary Conditions

$$k_{ix}x = k_{rx}x = k_{tx}x + \frac{\gamma\omega}{ic}\sin(\theta_t)x$$

$$\theta_i = \theta_r \qquad k_i\sin\theta_i = \tilde{k}_t\sin\tilde{\theta}_t$$

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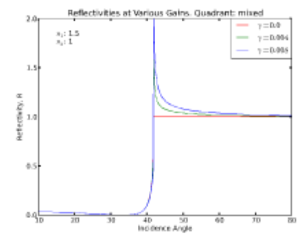
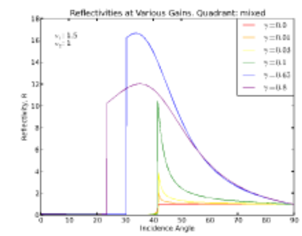
$$E_{ix} + E_{rx} = E_{tx} \qquad D_{iz} + D_{rz} = D_{tz}$$

Reflection Coefficients

$$\tilde{r} = \frac{(\epsilon_1 k'_{tz} - \epsilon'_2 k_{iz}) + i(\epsilon_1 k''_{tz} - \epsilon''_2 k_{iz})}{(\epsilon_1 k'_{tz} + \epsilon'_2 k_{iz}) + i(\epsilon_1 k''_{tz} + \epsilon''_2 k_{iz})}$$

$$R = \tilde{r}^*\tilde{r} = \frac{(\epsilon_1 k'_{tz} - \epsilon'_2 k_{iz})^2 + (\epsilon_1 k''_{tz} - \epsilon''_2 k_{iz})^2}{(\epsilon_1 k'_{tz} + \epsilon'_2 k_{iz})^2 + (\epsilon_1 k''_{tz} + \epsilon''_2 k_{iz})^2}$$





Which Quadrant?

$$\tilde{k^2}_{tz} = k_0^2\left(\epsilon'_2 - \epsilon_1\sin^2\theta_i + i\epsilon''_2\right)$$

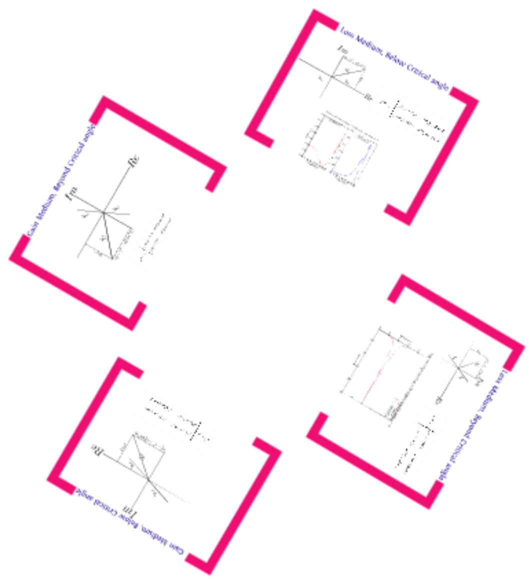
$$k'_{tz} = \pm\frac{k_0}{\sqrt{2}}\sqrt{\epsilon'_2 - \epsilon_1\sin^2\theta_i - \sqrt{(\epsilon'_2 - \epsilon_1\sin^2\theta_i)^2 - \epsilon''^2_2}}$$

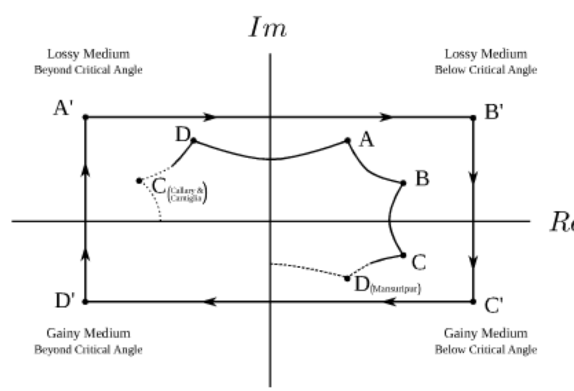
$$k''_{tz} = \pm\frac{k_0}{\sqrt{2}}\sqrt{\epsilon_1\sin^2\theta_i - \epsilon'_2 - \sqrt{(\epsilon'_2 - \epsilon_1\sin^2\theta_i)^2 - \epsilon''^2_2}}$$

1. Amplitude grows in propagation direction for gain medium, and decays in loss medium.

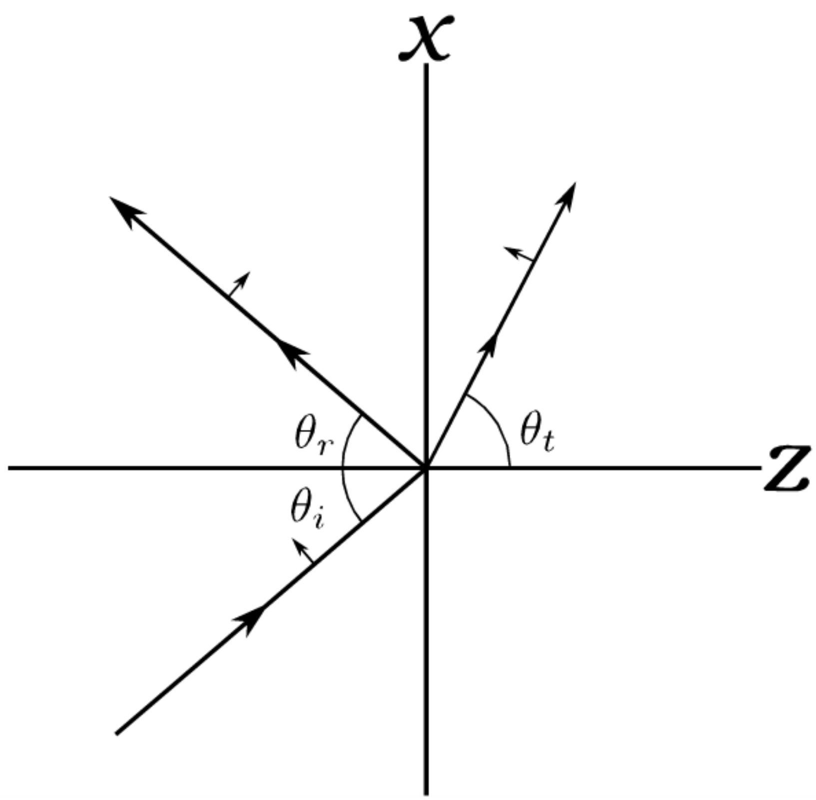
2. A non-evanescent wave (below the critical angle) must propagate away from the boundary.

3. An evanescent wave (beyond the critical angle) must decay away from the boundary.





Geometry



$$\mathbf{E}_i(\mathbf{x},t) = E_{0i}(\cos\theta_i\hat{\mathbf{x}} - \sin\theta_i\hat{\mathbf{z}})e^{i(k_{ix}x+k_{iz}z-\omega t)}$$

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Reflection Coefficients

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Which Quadrant?

$$\tilde{k^2}_{tz} = k_0^2\left(\epsilon_2' - \epsilon_1 \sin^2\theta_i + i\epsilon_2''\right)$$

$$k_{tz}' = \pm\frac{k_0}{\sqrt{2}}\sqrt{\epsilon_2' - \epsilon_1\sin^2\theta_i + \sqrt{(\epsilon_2' - \epsilon_1\sin^2\theta_i)^2 + \epsilon_2''^2}}$$

$$k_{tz}'' = \pm\frac{k_0}{\sqrt{2}}\sqrt{\epsilon_1\sin^2\theta_i - \epsilon_2' + \sqrt{(\epsilon_2' - \epsilon_1\sin^2\theta_i)^2 + \epsilon_2''^2}}$$

1. Amplitude grows in propagation direction for gain medium, and decays in loss medium.

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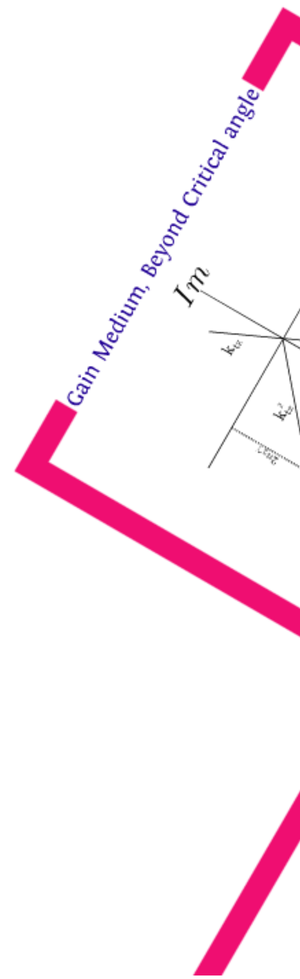
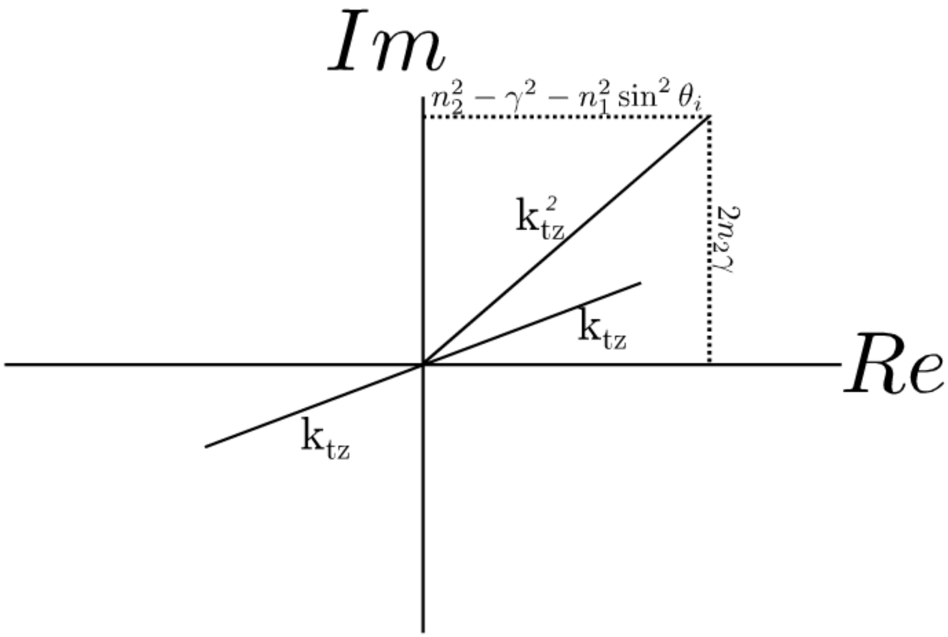
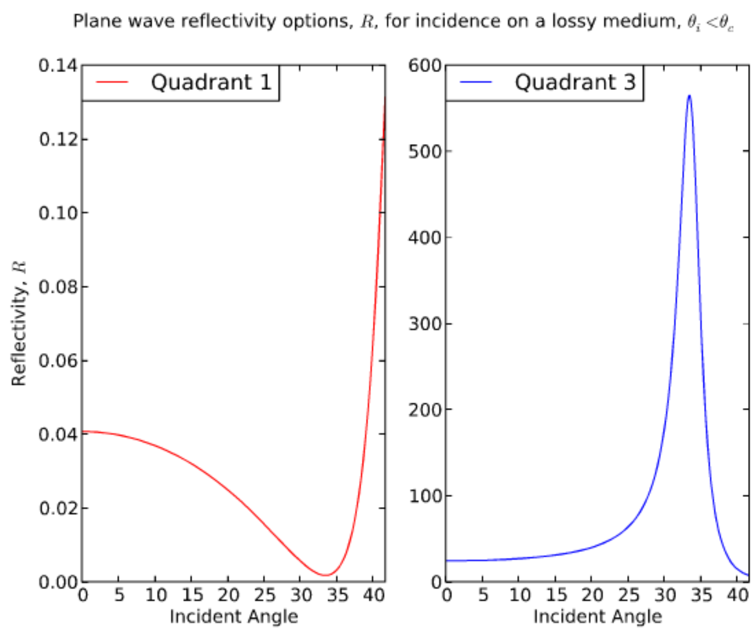


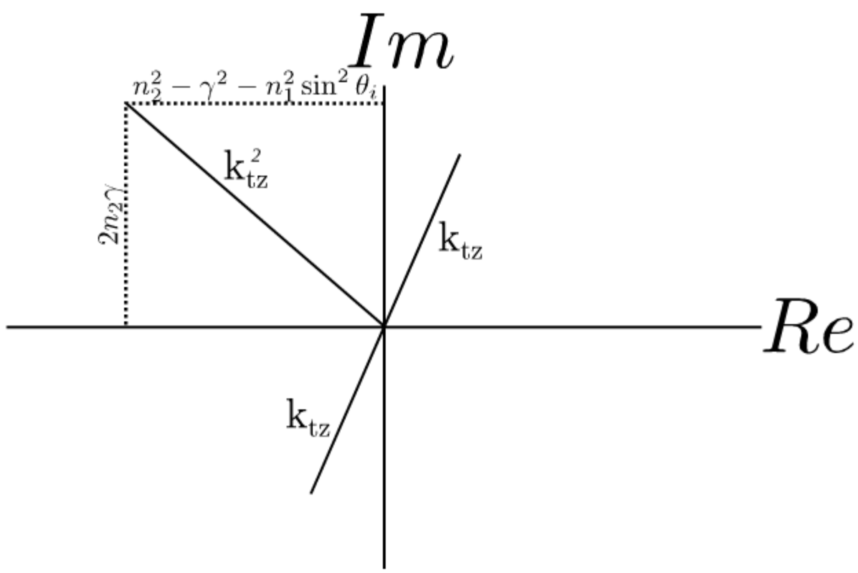
Loss Medium, Below Critical angle



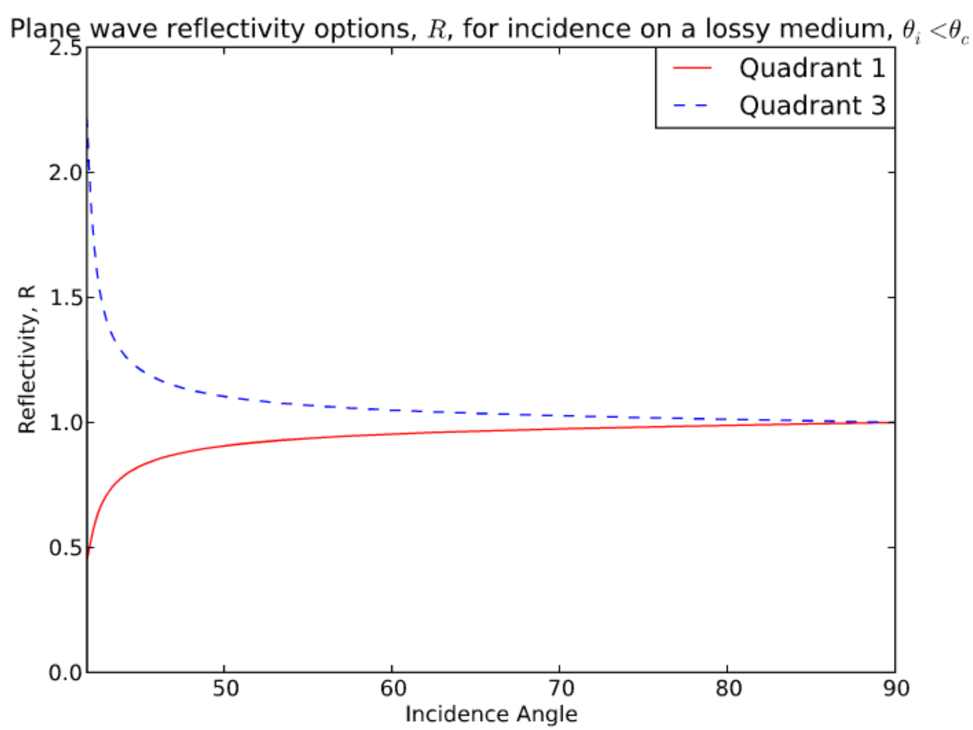
$$e^{i\tilde{k}_{tz}} = \begin{cases} e^{i|k'_{tz}|z - |k''_{tz}|z} & 1^{st} Quadrant \\ e^{-i|k'_{tz}|z + |k''_{tz}|z} & 3^{rd} Quadrant \end{cases}$$



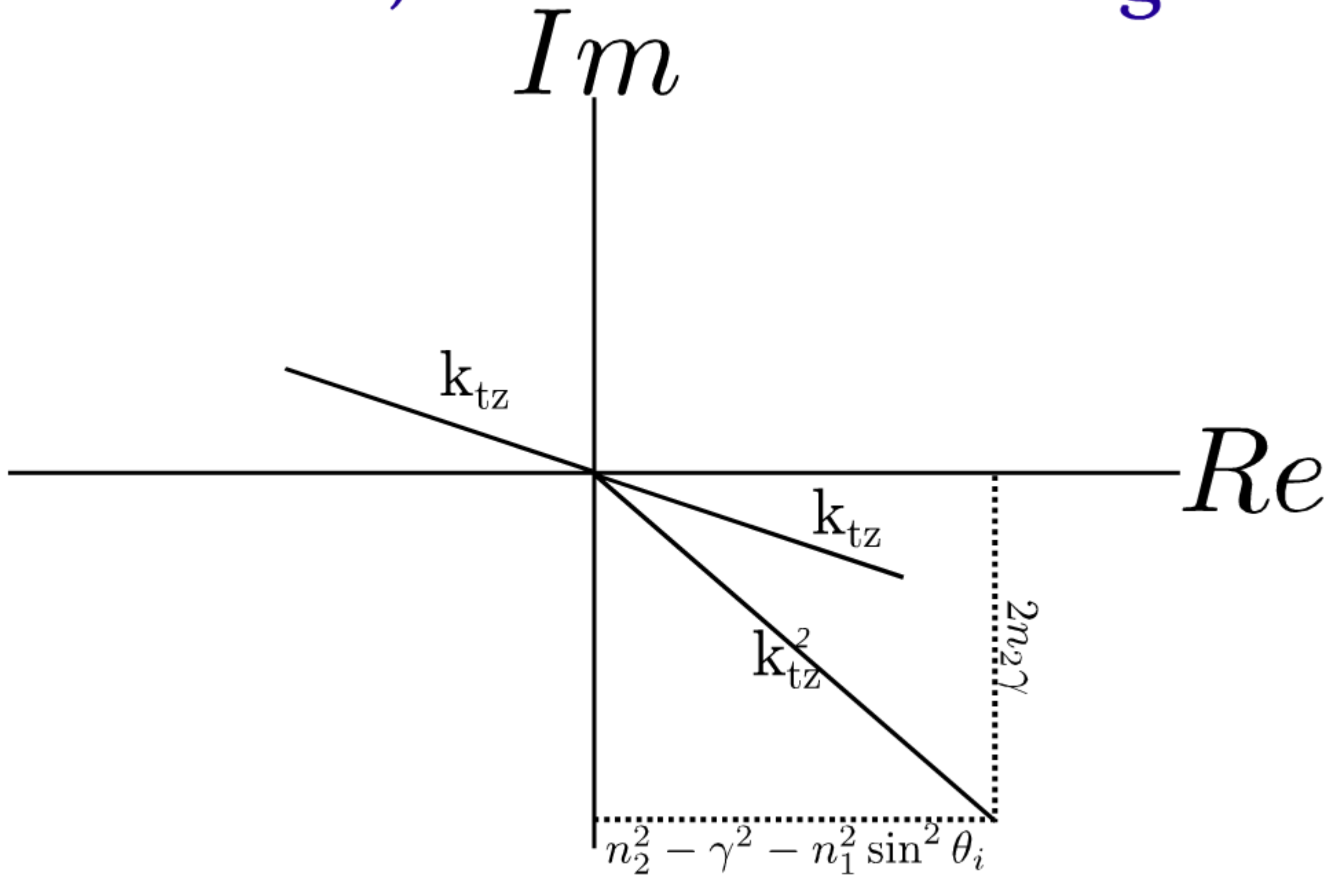
Loss Medium, Beyond Critical angle



$$e^{i\tilde{k}_{tz}} = \begin{cases} e^{i|k'_{tz}|z - |k''_{tz}|z} & 1^{st} Quadrant \\ e^{-i|k'_{tz}|z + |k''_{tz}|z} & 3^{rd} Quadrant \end{cases}$$

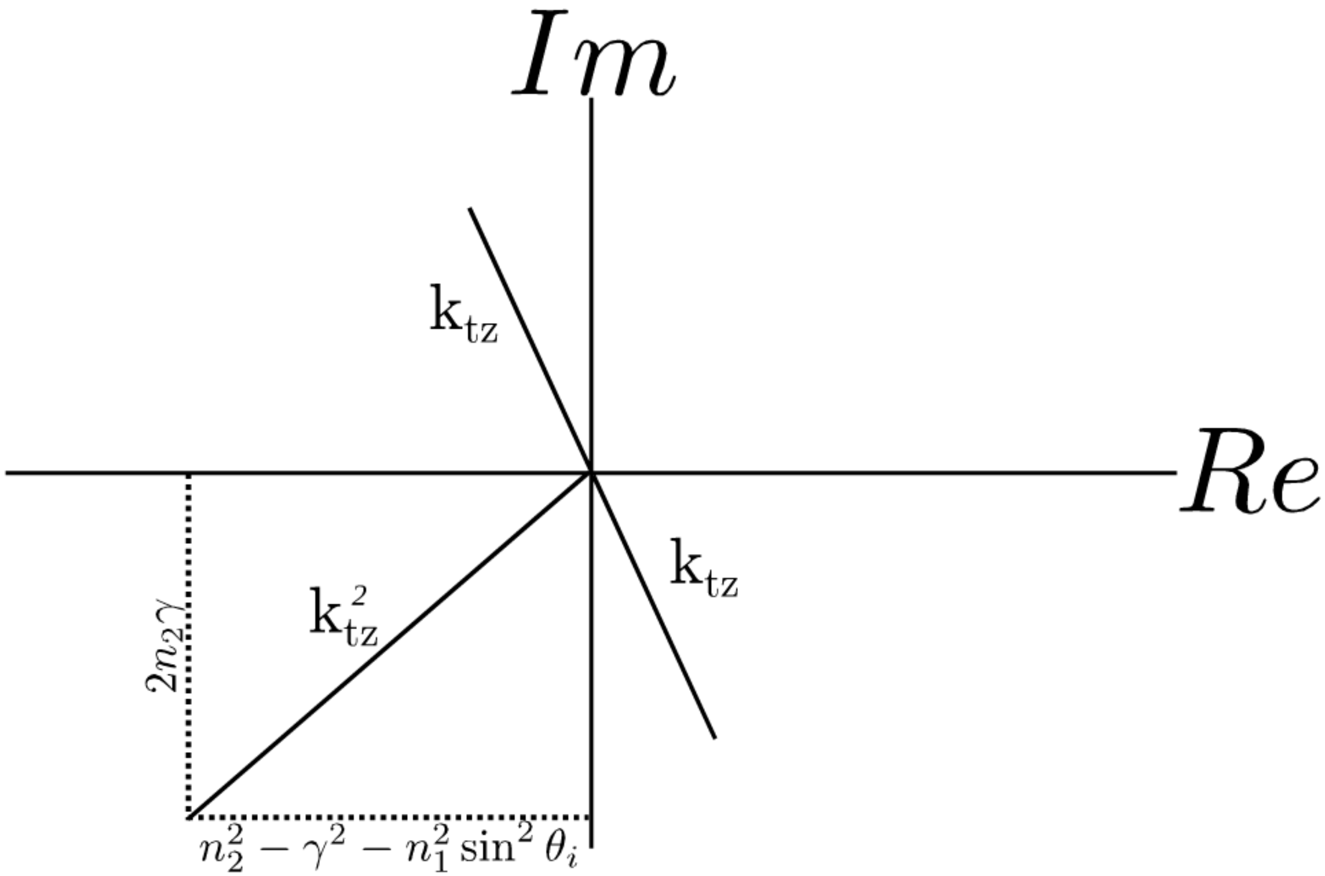


Gain Medium, Below Critical angle

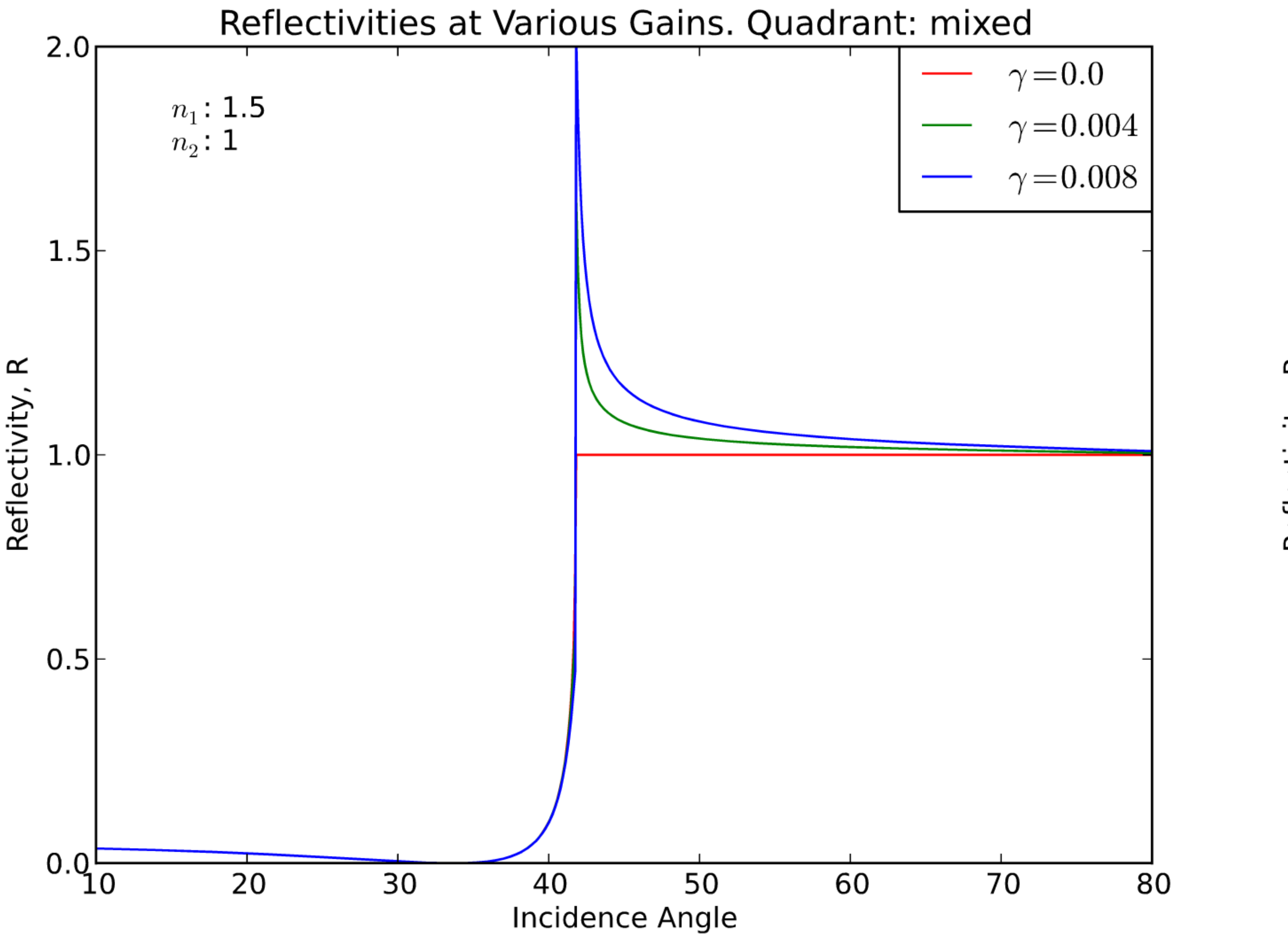


$$e^{i\tilde{k}_{tz}} = \begin{cases} e^{-i|k'_{tz}|z-|k''_{tz}|z} & 2^{nd} Quadrant \\ e^{i|k'_{tz}|z+|k''_{tz}|z} & 4^{th} Quadrant \end{cases}$$

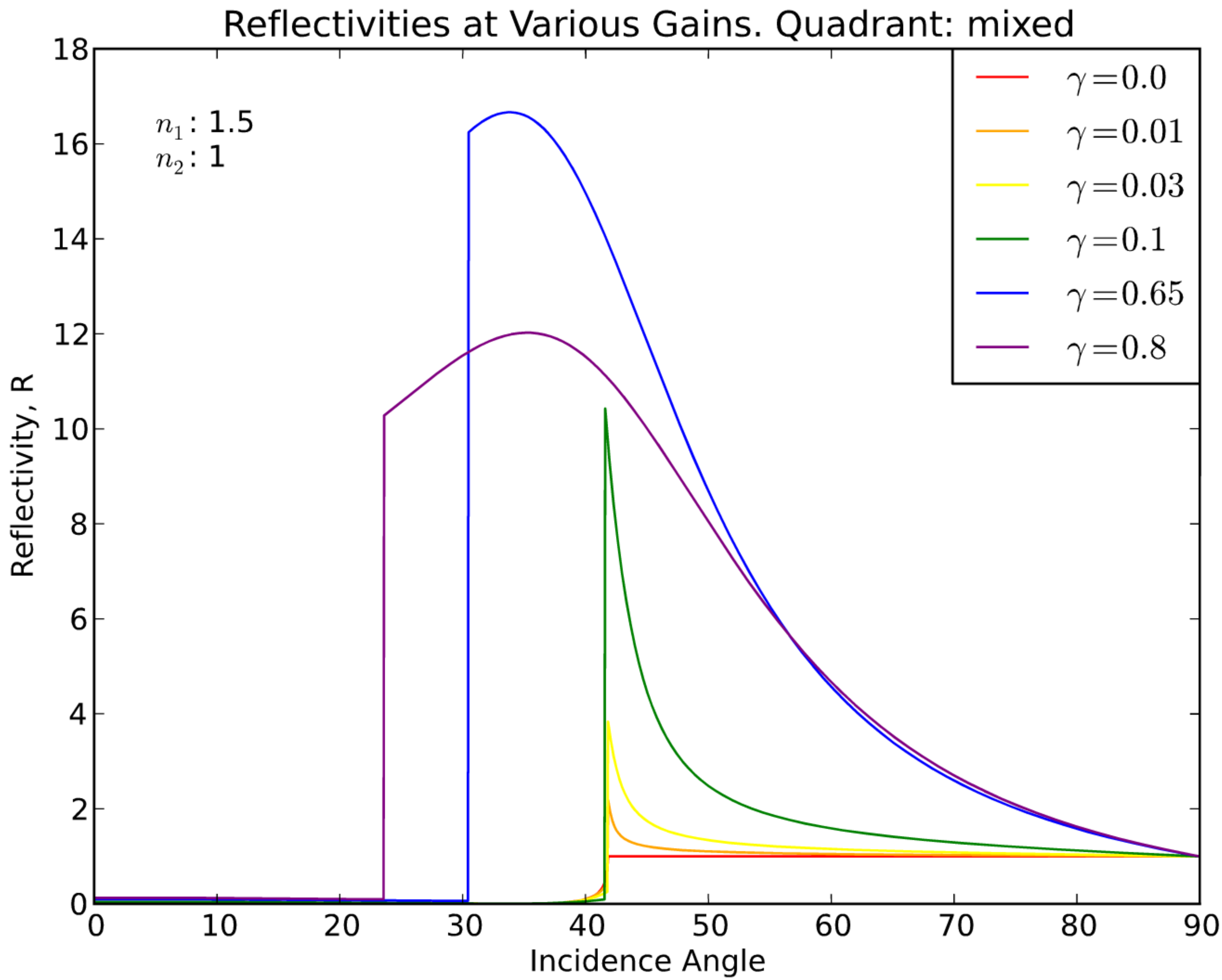
Gain Medium, Beyond Critical angle



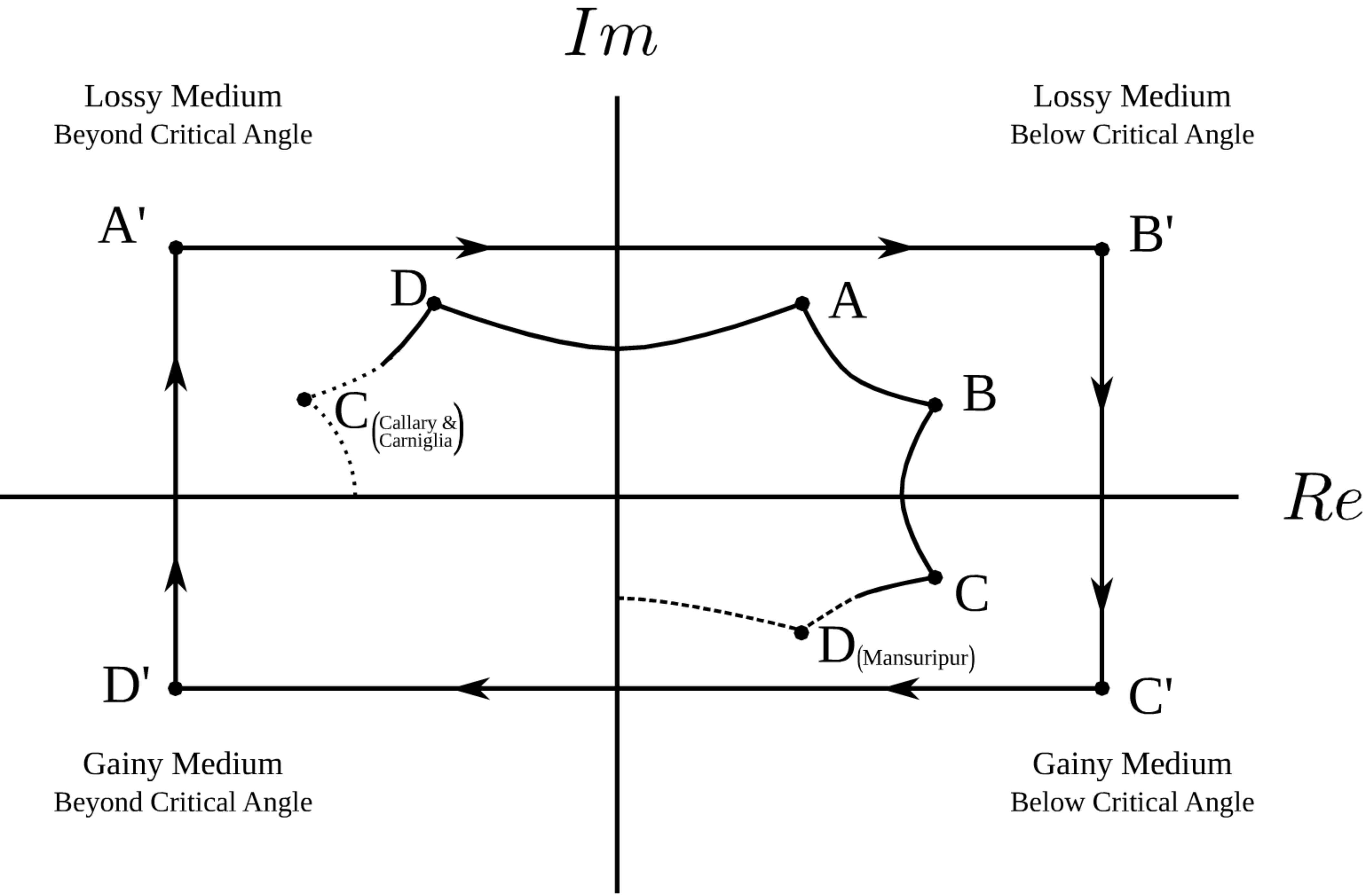
$$e^{i\tilde{k}_{tz}} = \begin{cases} e^{-i|k'_{tz}|z-|k''_{tz}|z} & 2^{nd} Quadrant \\ e^{i|k'_{tz}|z+|k''_{tz}|z} & 4^{th} Quadrant \end{cases}$$



Reflectivities at Various Gains. Quadrant: mixed
n_1: 1.5
n_2: 1
$\gamma = 0.0$
$\gamma = 0.004$
$\gamma = 0.008$
Reflectivity, R
Incidence Angle
2.0
1.5
1.0
0.5
0.0
10
20
30
40
50
60
70
80



Reflectivities at Various Gains. Quadrant: mixed
n_1: 1.5
n_2: 1
$\gamma = 0.0$
$\gamma = 0.01$
$\gamma = 0.03$
$\gamma = 0.1$
$\gamma = 0.65$
$\gamma = 0.8$
Reflectivity, R
Incidence Angle
0
2
4
6
8
10
12
14
16
18
0
10
20
30
40
50
60
70
80
90

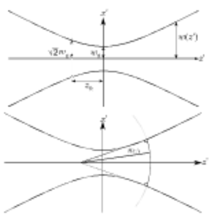


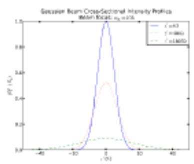
Im
Lossy Medium
Beyond Critical Angle
Lossy Medium
Below Critical Angle
A'
B'
D
A
B
C
(Callary & Carniglia)
Re
C
D
(Mansuripur)
D'
C'
Gainy Medium
Beyond Critical Angle
Gainy Medium
Below Critical Angle

Finite Beam

A Gaussian Beam

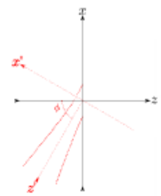
$$E(\mathbf{x}') = E_0 \frac{w_0}{w^2(z')}\exp\left[-\frac{x'^2+y'^2}{w^2(z')} + ik\left(z' - \frac{x'^2+y'^2}{2R(z')}\right) + i\xi(z')\right]$$





The Fourier Transform Method

$$E_i(x) \approx \frac{E_0}{\sqrt{1+\frac{x^2\sin^2\phi}{z_R^2}}}\exp\left[\frac{-x^2\cos^2\phi}{w_0^2\left(1+\frac{x^2\sin^2\phi}{z_R^2}\right)} - ikx\sin\phi - ik\frac{x\cos^2\phi}{2\sin\phi\left(1+\frac{z_R^2}{x^2\sin^2\phi}\right)} + i\arctan\left(\frac{x\sin\phi}{z_R}\right)\right]$$



Transform to plane waves

$$E_i(k_x) = \int_{-\infty}^{\infty} E_i(x)e^{-ik_x x}\,\mathrm{d}x$$

Reflect each plane wave

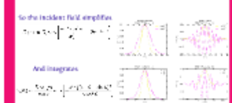
$$E_r(k_x) = E_i(k_x)\tilde{r}(k_x)$$

Analytical

The transform cannot be computed analytically

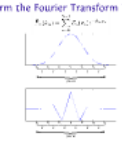
So the incident field simplifies

And integrates



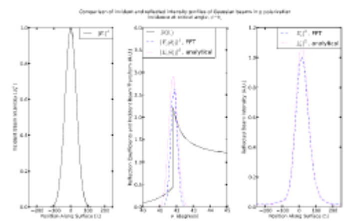
DFT

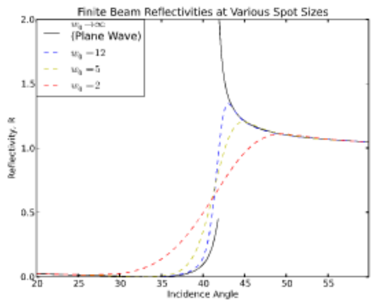
Perform the Fourier Transform Discretely

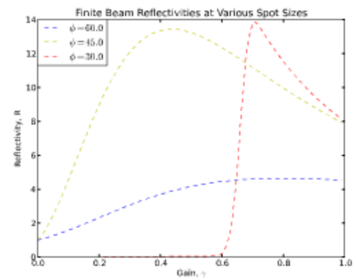


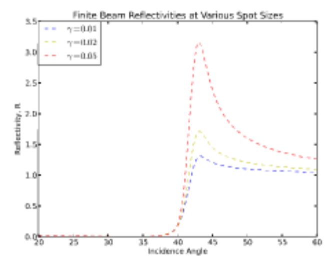
Inverse transform

$$E_r(x,z) = \int_{-\infty}^{\infty} E_r(k_x)e^{i(k_x x + k_z z)}\,\mathrm{d}k_x$$



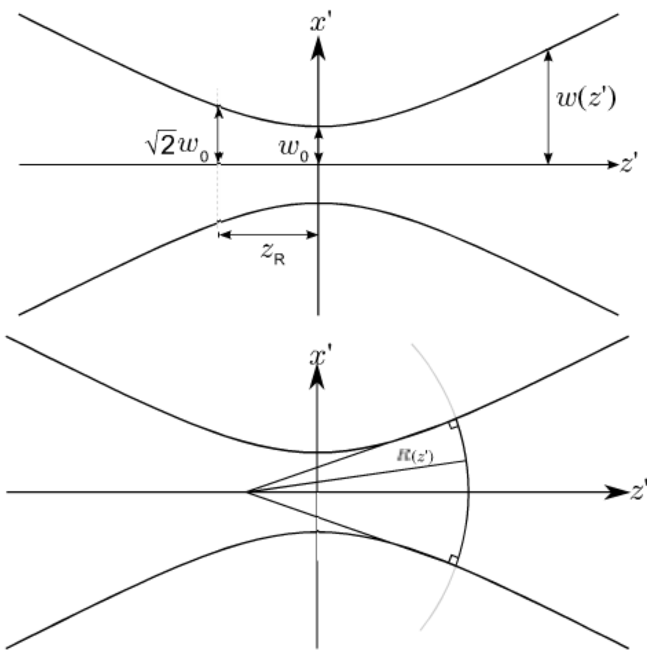


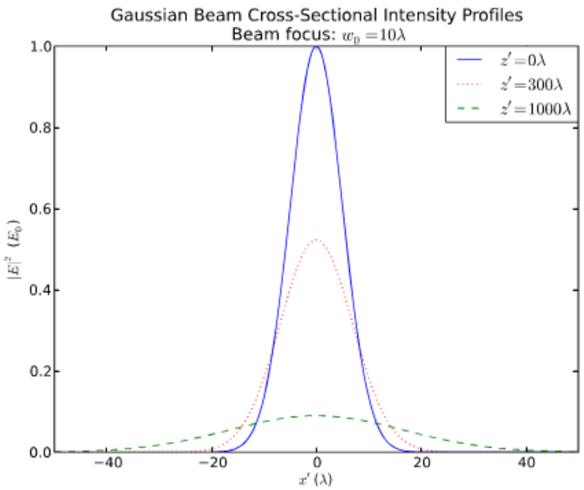




A Gaussian Beam

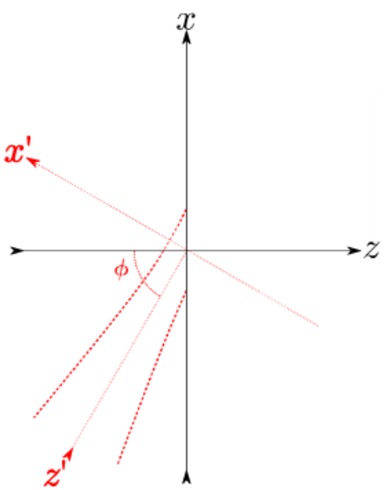
$$E(\mathbf{x}') = E_0 \frac{w_0}{w^2(z')} \exp\left[-\frac{x'^2+y'^2}{w^2(z')} + ik\left(z' - \frac{x'^2+y'^2}{2R(z')}\right) + i\xi(z')\right]$$





The Fourier Transform Method

$$E_i(x) \approx \frac{E_0}{\sqrt{1+\frac{x^2\sin^2\phi}{z_R^2}}} \exp\left[\frac{-x^2\cos^2\phi}{w_0^2\left(1+\frac{x^2\sin^2\phi}{z_R^2}\right)} + ikx\sin\phi - ik\frac{x\cos^2\phi}{2\sin\phi\left(1+\frac{z_R^2}{x^2\sin^2\phi}\right)} + i\arctan\left(\frac{x\sin\phi}{z_R}\right)\right]$$



Transform to plane waves

$$E_i(k_x) = \int_{-\infty}^{\infty} E_i(x)e^{-ik_x x}\,\mathrm{d}x$$

Reflect each plane wave

$$E_r(k_x) = E_i(k_x)\tilde{r}(k_x)$$

Inverse transform

$$E_r(x,z) = \int_{-\infty}^{\infty} E_r(k_x)e^{i(k_x x + k_z z)}\,\mathrm{d}k_x$$

Analytical

The transform cannot be computed analytically

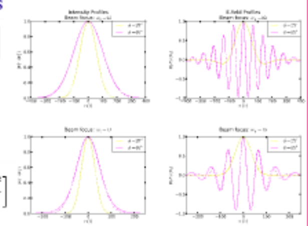
$$\arctan\left(\frac{z'}{z_R}\right) \approx 0 \qquad x^2/z_R^2 \ll 1$$

So the incident field simplifies

$$E_i(x) \approx E_0 \exp\left[\frac{x^2\cos^2\phi}{w_0^2} + ikx\sin\phi\right]$$

And integrates

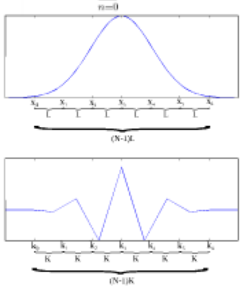
$$E_i(\theta_i) \approx \frac{E_0 w_0\sqrt{\pi}}{\cos\phi}\exp\left[-\frac{k^2 w_0^2(\sin\phi - \sin\theta_i)^2}{4\cos^2\phi}\right]$$



DFT

Perform the Fourier Transform Discretely

$$\tilde{F}_m(k_m) = \sum_{n=0}^{N-1} E_n(x_n)e^{-ik_m x_n}$$



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And integrates

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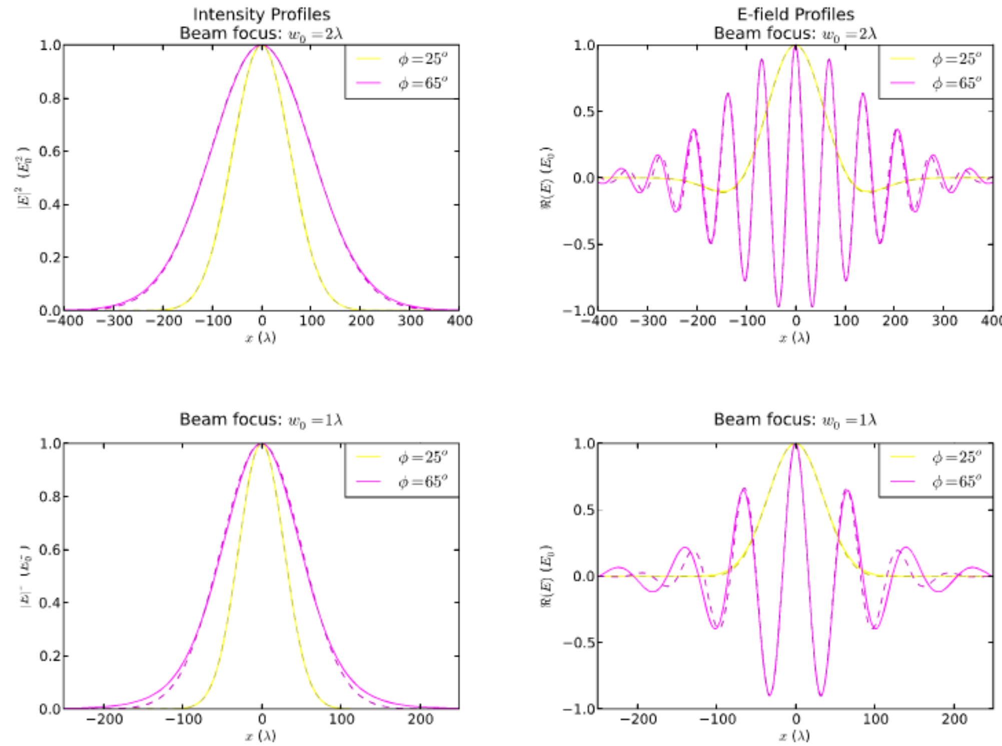
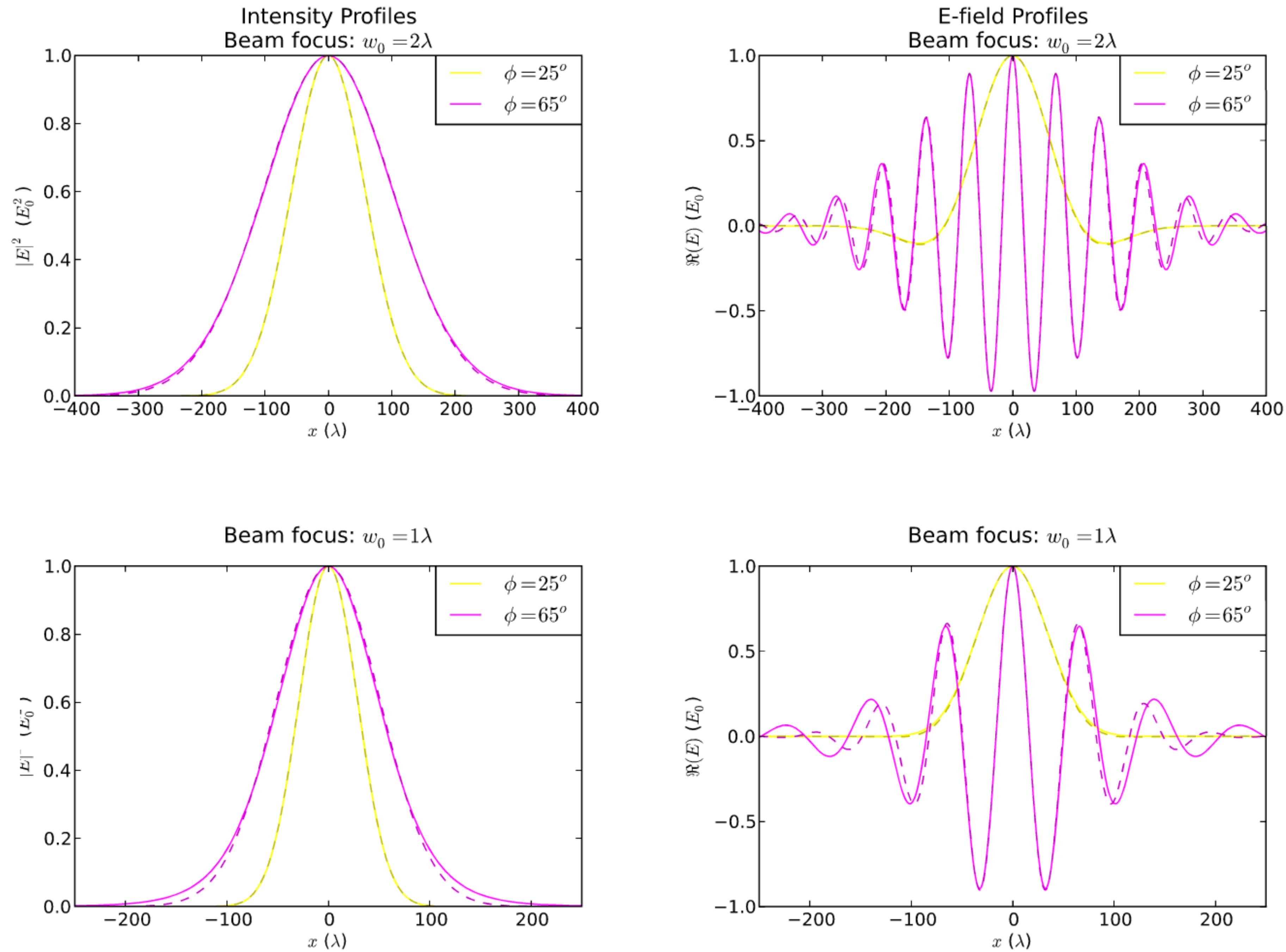


s

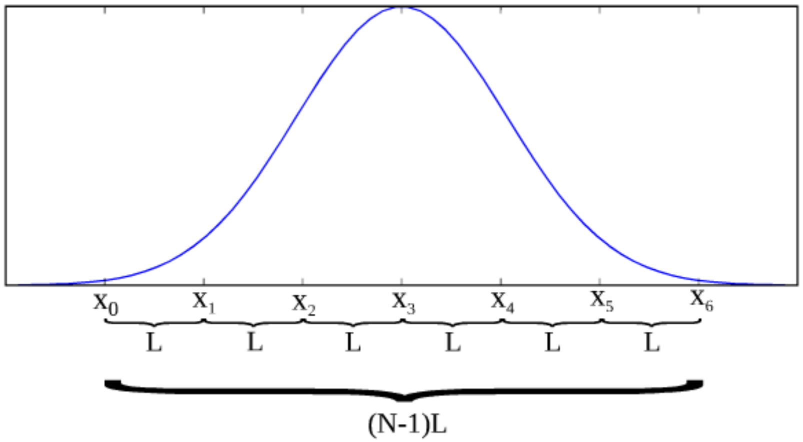


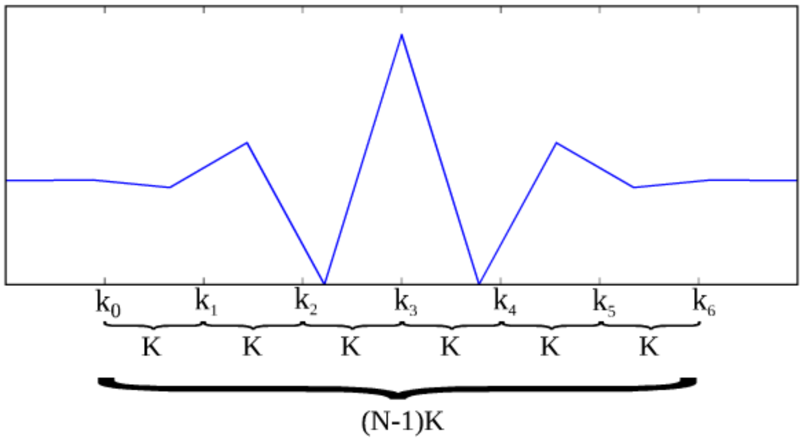
Intensity Profiles
Beam focus: $w_0 = 2\lambda$
E-field Profiles
Beam focus: $w_0 = 2\lambda$
Beam focus: $w_0 = 1\lambda$
Beam focus: $w_0 = 1\lambda$
$\phi = 25^o$
$\phi = 65^o$
$|E|^2$ (E_0^2)
$\Re(E)$ (E_0)
x (λ)
1.0
0.8
0.6
0.4
0.2
0.0
0.5
−0.5
−1.0
−400 −300 −200 −100 0 100 200 300 400
−200 −100 0 100 200

DFT

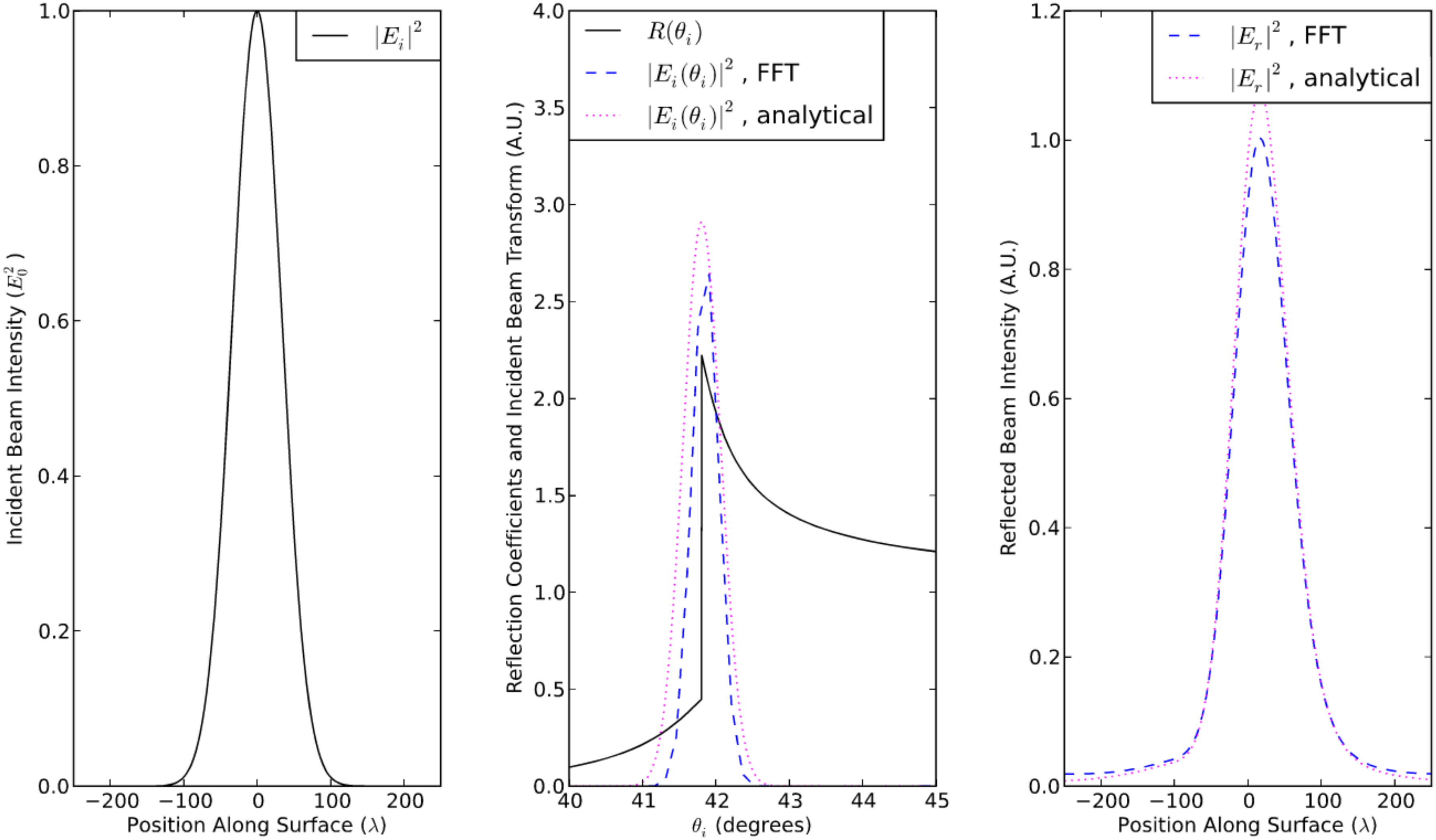
Perform the Fourier Transform Discretely

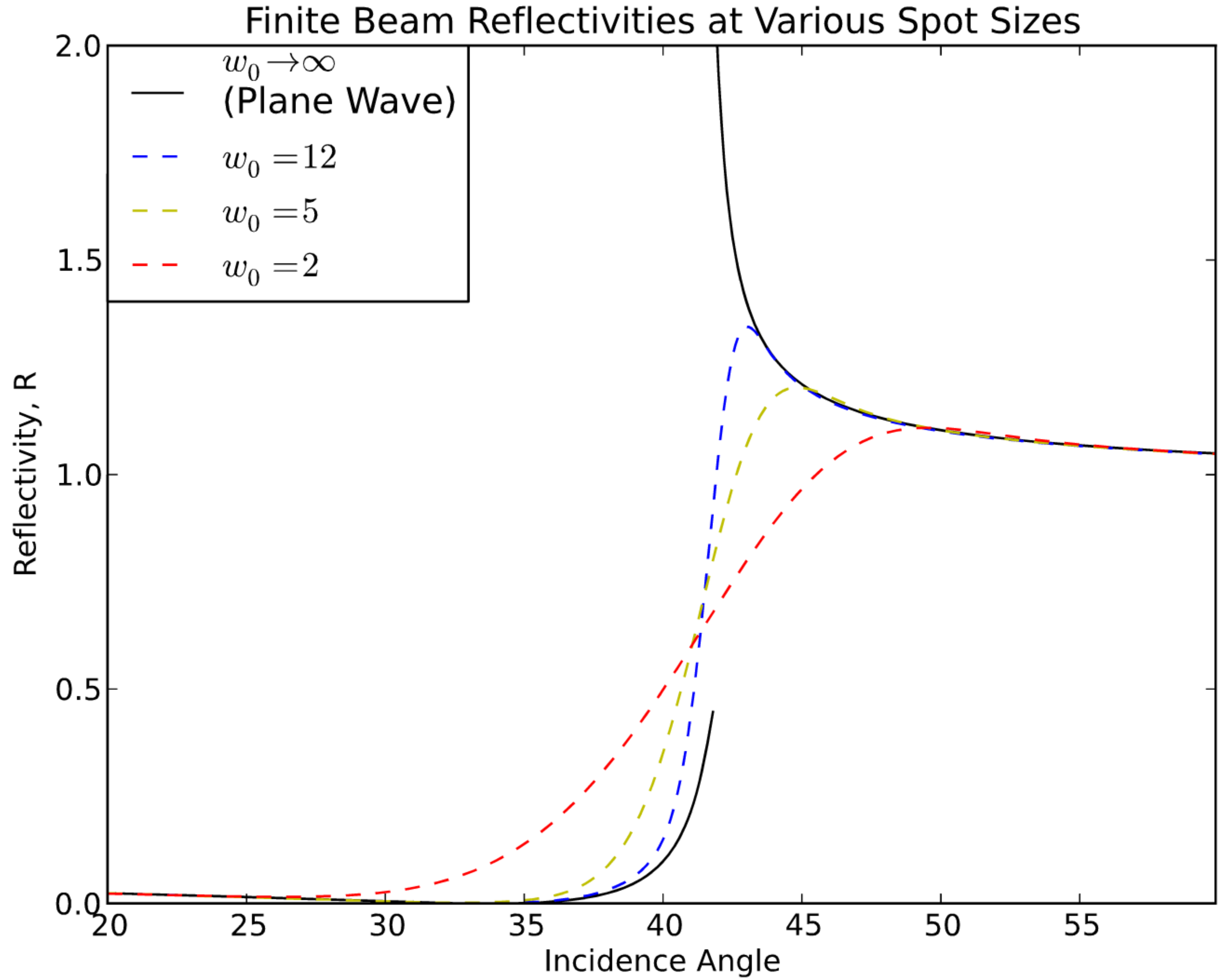
$$\tilde{F}_m(k_m) = \sum_{n=0}^{N-1} E_n(x_n) e^{-ik_m x_n}$$



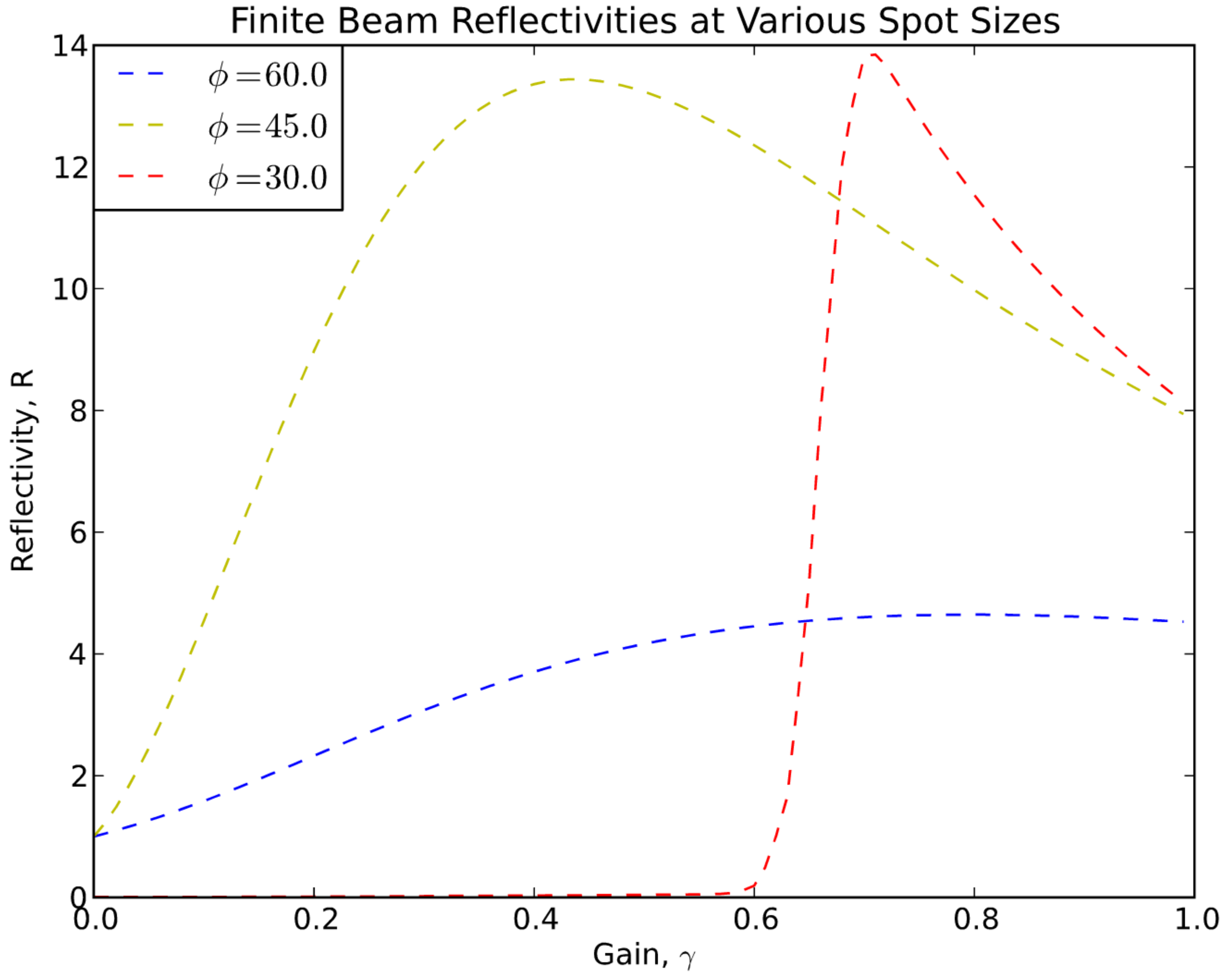


Comparison of incident and reflected intensity profiles of Gaussian beams in p polarization
Incidence at critical angle, $\phi = \theta_c$

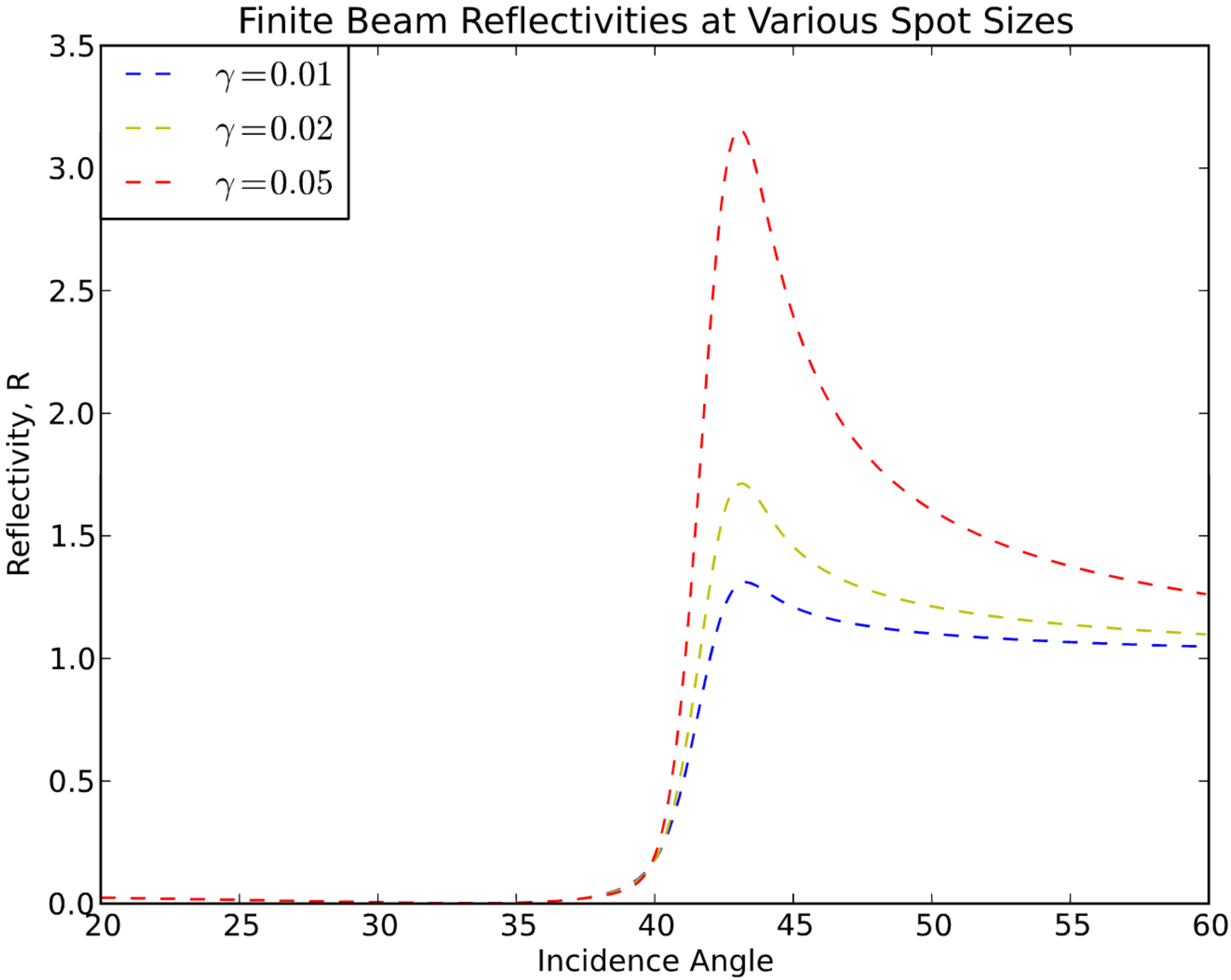




Finite Beam Reflectivities at Various Spot Sizes
$w_0 \rightarrow \infty$
(Plane Wave)
$w_0 = 12$
$w_0 = 5$
$w_0 = 2$
Reflectivity, R
Incidence Angle
0.0
0.5
1.0
1.5
2.0
20
25
30
35
40
45
50
55

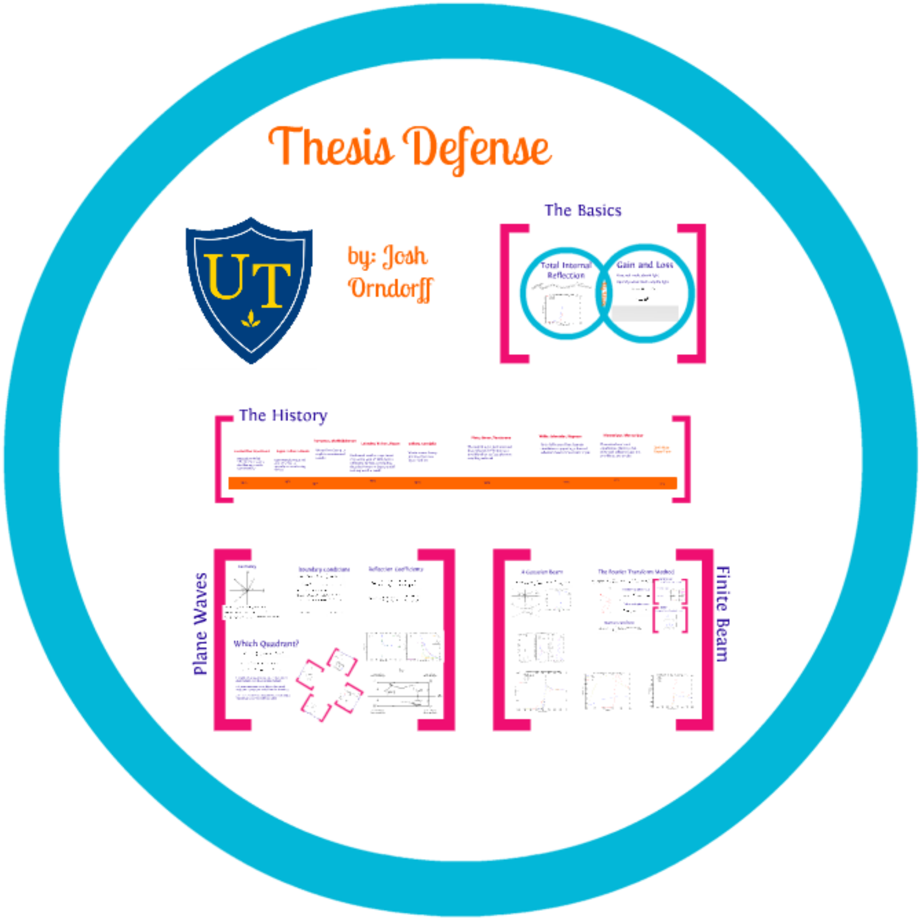


Finite Beam Reflectivities at Various Spot Sizes
$\phi = 60.0$
$\phi = 45.0$
$\phi = 30.0$
Reflectivity, R
Gain, γ
0
2
4
6
8
10
12
14
0.0
0.2
0.4
0.6
0.8
1.0



Finite Beam Reflectivities at Various Spot Sizes
$\gamma = 0.01$
$\gamma = 0.02$
$\gamma = 0.05$
Reflectivity, R
Incidence Angle
0.0
0.5
1.0
1.5
2.0
2.5
3.0
3.5
20
25
30
35
40
45
50
55
60

Amplified Total Internal Reflection at the Surface of a Gain Medium



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